

Lecture 5: Labour Economics and Wage-Setting Theory

Spring 2017

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Literature: Chapter 7 Cahuc-Carcillo-Zylberberg: 435-445

Topics

- **Weakly efficient bargaining**
- **Strongly efficient bargaining**
- **Wage dispersion**
- **Bargaining over working time**
- **~~Kpuf gt u'cpf 'qwwuf gt u''~~**

Efficient contracts

- **Bargaining over the wage only and letting employers determine employment (right to manage) is not efficient.**
- **An efficient solution can be found by bargaining over both the wage and employment.**

$$\begin{aligned} \text{Max}_{w, L} \quad & [R(L) - wL]^{1-\gamma} [\nu(w) - \nu(\bar{w})]^\gamma L^\gamma \\ \text{s.t.} \quad & 0 \leq L \leq N \quad \text{and} \quad w \geq \bar{w} \end{aligned}$$

Interior solution

$$(1 - \gamma) \frac{R'(L) - w}{R(L) - wL} + \frac{\gamma}{L} = 0 \quad (\text{I})$$

$$-(1 - \gamma) \frac{L}{R(L) - wL} + \frac{\gamma \nu'(w)}{\nu(w) - \nu(\bar{w})} = 0 \quad (\text{II})$$

Eliminate γ between the two equations to get

$$w - R'(L) = \frac{\nu(w) - \nu(\bar{w})}{\nu'(w)} \quad (\text{III})$$

This is the equation of a **contract curve** (Pareto-efficient combinations of w, L) connecting tangency points of indifference and isoprofit curves.

The same equation would be obtained by maximising

$$L[\nu(w) - \nu(\bar{w})] \quad \text{s.t.} \quad \pi = \bar{\pi}$$

Differentiation of the contract curve equation gives:

$$\frac{dw}{dL} = \frac{R''(L)}{\nu''(w)[w - R'(L)]}$$

$\gamma = 0 \Rightarrow R'(L) = w$ according to (I)

$R'(L) = w \Rightarrow \nu(w) = \nu(\bar{w})$ and $w = \bar{w}$ according to (III)

Hence the contract curve starts on the labour demand schedule at $w = \bar{w}$

If $w > R'(L)$ and workers are risk averse, i.e.

$\nu'' < 0$, then $dw / dL > 0$ for $w > R'(L)$.

$\gamma = 0$ gives the competitive level of employment $L = L(\bar{w})$

With $\gamma > 0$, the union uses its bargaining power to raise both the wage and employment over the competitive levels.

If workers are risk-neutral, then $\nu'' = 0$ and $\frac{dw}{dL} \rightarrow \infty$. Hence the contract curve is vertical. Employment is at the competitive level.

Overemployment if workers are risk-averse – “weak efficiency” as

$R'(L) < \bar{w}$ due to employment being higher than L_c defined by $R'(L_c) = \bar{w}$

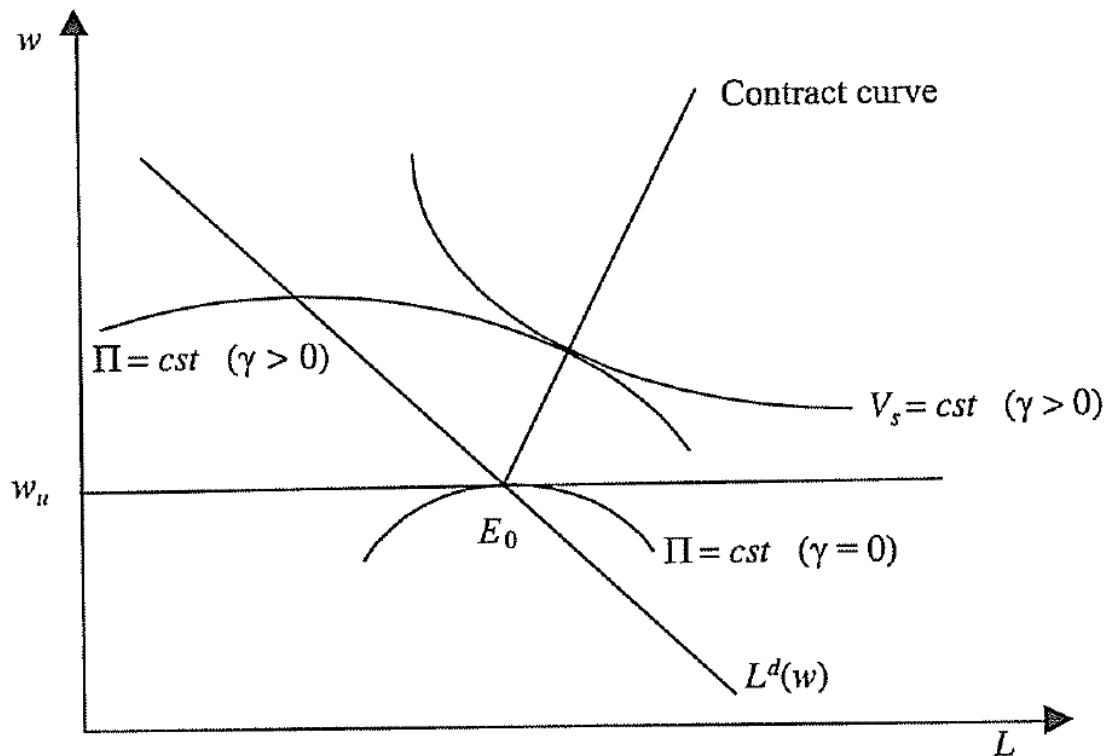


FIGURE 7.6

The model of bargaining over wages and employment.

Strongly efficient contracts

- Efficiency gain for union if utility of employed and unemployed are equated
- Incentive to bargain with firm over unemployment benefit paid by the firm

Union objective

$$L\nu(w) + (N - L)\nu(b + \bar{w})$$

Firm profit

$$\pi = R(L) - wL - (N - L)b$$

$$\text{Max}_{w, b} L\nu(w) + (N - L)\nu(b + \bar{w})$$

$$\text{s.t. } \pi = \pi_0$$

$$\text{Max}_{w, b} L\nu(w) + (N - L)\nu(b + \bar{w}) + \lambda [R(L) - wL - (N - L)b - \pi_0]$$

FOC

$$L\nu'(w) - \lambda L = 0$$

$$(N - L)\nu'(b + \bar{w}) - \lambda(N - L) = 0$$

$$\nu'(w) = \lambda$$

$$\nu'(b + \bar{w}) = \lambda$$

Hence:

$$\nu'(w) = \nu'(b + \bar{w})$$

$$w = b + \bar{w}$$

- **Pareto efficiency requires a wage for the employed that is equal to the income as unemployed.**
- **The firm pays a benefit b to all unemployed.**
- **It pays a wage $\bar{w} + b$ to the employed.**
- **Employment does not matter to the union, since members are insured against unemployment.**

The bargaining problem

$$\text{Max}_b \left[R(L^*) - \bar{w}L^* - bN \right]^{1-\gamma} \left[\nu(\bar{w} + b) - \nu(\bar{w}) \right]^\gamma$$

FOC:

$$\frac{\nu(\bar{w} + b) - \nu(\bar{w})}{\nu'(\bar{w} + b)} = \frac{\gamma}{1 - \gamma} \frac{\left[R(L^*) - \bar{w}L^* - bN \right]}{N}$$

$$\text{with } w = \bar{w} + b$$

$$R'(L^*) = \bar{w}$$

- **Employment equals the competitive level**
- **Union members appropriate a share of the firm's profit without this having negative effects on employment**

Diagrammatical illustration

Indifference curves:

$$v_s = v(w)$$

$$v_1 dw = 0$$

$$\frac{v_1 dw}{dL} = 0$$

$$\frac{dw}{dL} = 0$$

The indifference curves are horizontal lines.

Isoprofit curve

$$\pi = R(L) - \bar{w}L - bN = R(L) - \bar{w}L - N(w - \bar{w})$$

$$d\pi = 0 = R'(L)dL - \bar{w}dL - Ndw$$

$$\frac{dw}{dL} = \frac{R'(L) - \bar{w}}{N}$$

- Tangency points between isoprofit curves and indifference curves give a vertical contract curve (at the competitive level of employment)
- Bargaining over wages, employment and unemployment benefits from firms is strongly efficient.

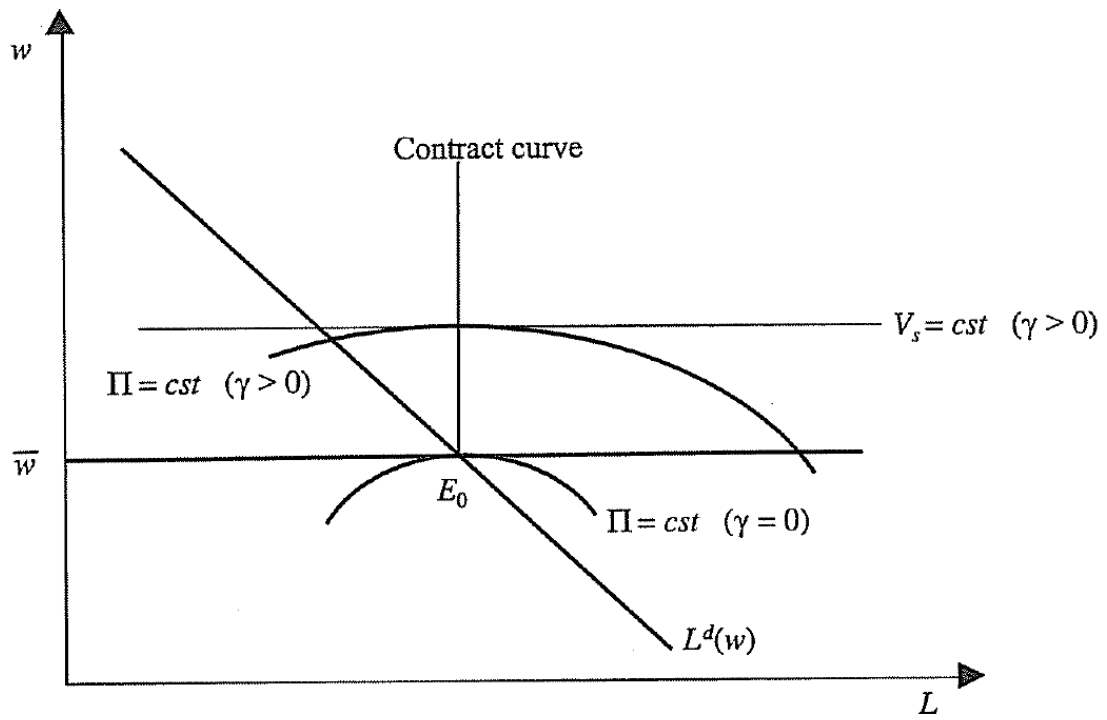


FIGURE 7.7
The strongly efficient bargaining model.

Collective bargaining and wage dispersion

- **Heterogeneous workers**
- **Collective bargaining reduces wage dispersion**
- **Two types of workers, indexed by $i = 1, 2$**
- **Revenue of the firm = $R(L_1, L_2)$**
- **Type -1 workers are more productive with a higher reservation wage $\bar{w}_1 > \bar{w}_2$**
- **N_i workers of type i in the firm's labour pool**
- **The union utility function**

$$v_s = \sum_{i=1}^2 L_i v(w_i) + (N_i - L_i) v(\bar{w}_i + b_i) \quad L_i \leq N_i$$

- **Strongly efficient bargaining over employment, wages and unemployment benefits**
- **Optimal contract implies $w_i = \bar{w}_i + b_i$**

Bargaining problem

$$\text{Max}_{b_1, b_2, L_1, L_2} \left[R(L_1, L_2) - \sum_{i=1}^2 (\bar{w}_i L_i + b_i N_i) \right]^{1-\gamma} \left[\sum_{i=1}^2 N_i \{ \nu(\bar{w}_i + b_i) - \nu(\bar{w}_i) \} \right]^\gamma$$

$$\text{s.t. } 0 \leq L_i \leq N_i \quad i = 1, 2$$

FOCs

$$(11) \quad \frac{\partial R(L_1, L_2)}{\partial L_i} = \bar{w}_i$$

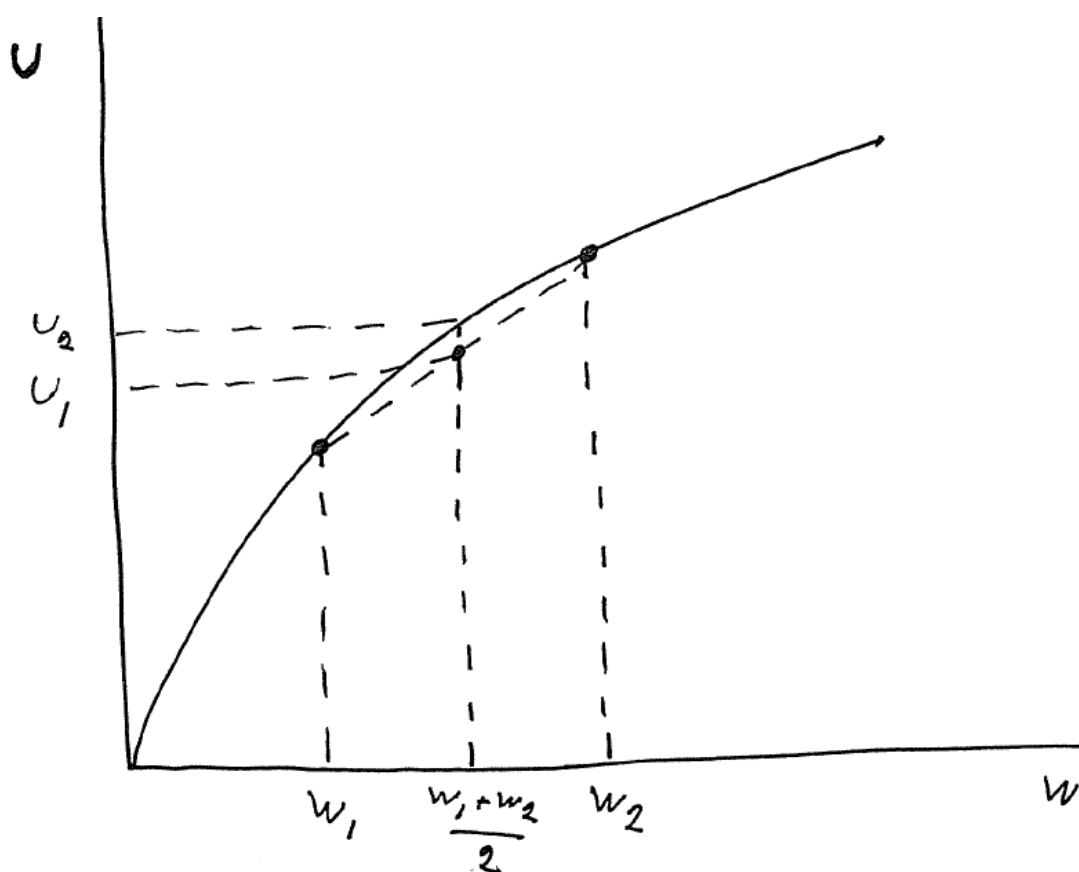
$$(12) \quad \nu'(\bar{w}_i + b_i) = \frac{1-\gamma}{\gamma} \frac{\left[\sum_{i=1}^2 N_i [\nu(\bar{w}_i + b_i) - \nu(\bar{w}_i)] \right]}{\left[R(L_1, L_2) - \sum_{i=1}^2 (\bar{w}_i L_i + b_i N_i) \right]}$$

- **Equation (11): Productive efficiency, i.e. the marginal productivity of each type of worker equals the reservation wage. This implies the competitive level of employment.**
- **Equation (12): RHS is independent of i . Hence the same wage for the two types of labour.**
- **Wage equality follows from the assumption of a utilitarian union and identical preferences.**

$$\frac{N_1}{N_1 + N_2} \nu(w_1) + \frac{N_2}{N_1 + N_2} \nu(w_2) \leq \nu \left[\frac{N_1}{N_1 + N_2} w_1 + \frac{N_2}{N_1 + N_2} w_2 \right]$$

Because of concavity the union is better off with a wage

$$\frac{N_1}{N_1 + N_2} w_1 + \frac{N_2}{N_1 + N_2} w_2 \text{ for everyone than with separate wages } w_1 \text{ and } w_2.$$



$$U_2 > U_1$$

Two-stage bargaining over employment (Manning 1987)

Stage 1: Bargaining over the wage

Stage 2: Bargaining over employment

Different bargaining strengths in the two negotiations

Bargaining over employment (given the wage)

$$\text{Max}_L \quad [R(L) - wL]^{1-\gamma_L} [\nu(w) - \nu(\bar{w})]^{\gamma_L} L^{\gamma_L} \quad \text{s.t. } 0 \leq L \leq N$$

The solution gives $L = L(\gamma_L, \bar{w}, w)$

Bargaining over the wage (takes the outcome of second-stage bargaining over employment into account)

$$\text{Max}_w \quad [R(L) - wL]^{1-\gamma} [\nu(w) - \nu(\bar{w})]^{\gamma} L^{\gamma}$$

$$\text{s.t. } L = \hat{L}(\gamma_L, \bar{w}, w) \quad \text{and} \quad w \geq \bar{w}$$

Different cases

- $\gamma_L = 0$ and $\gamma > 0$ gives the **right-to-manage model**
- $\gamma_L = \gamma$ gives **(weakly) efficient bargain model**
- **Otherwise solution on neither labour-demand schedule nor contract curve**

Motivations

- **Efficient bargaining is complex**
- **Wage bargaining precedes employment bargaining**
- **Wage bargaining is often at more centralised level**
- **Strongly efficient bargaining is improbable because of moral hazard problems: unemployed being fully insured will not search effectively for jobs**
 - **argument for partial insurance**
 - **individual firm (sector) offering full insurance would be swamped by labour inflow**
- **One does not find many examples of contracts with unemployment benefits paid by firms**
- **Unclear empirical results on right-to-manage model and (weakly efficient) bargaining**

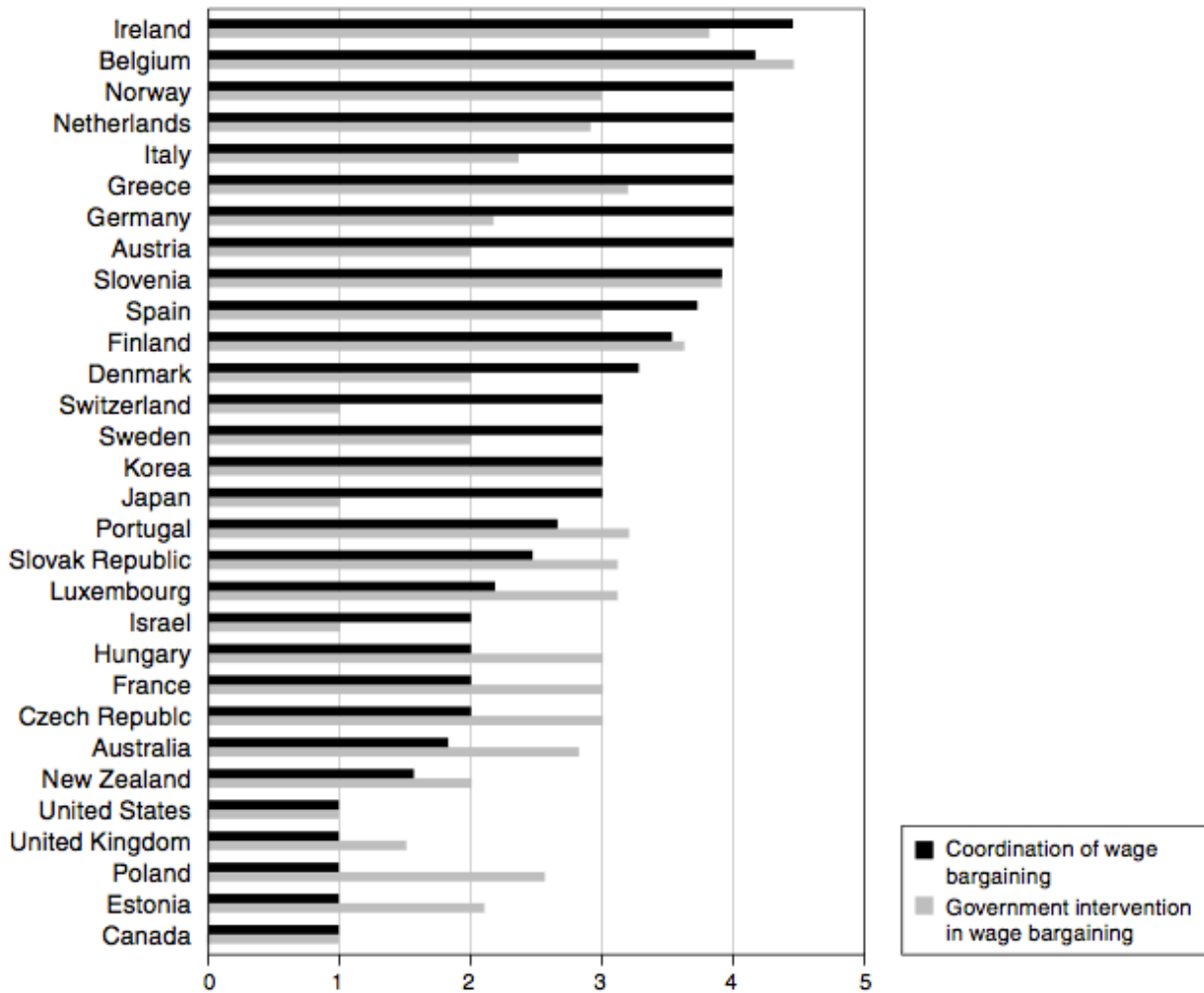


FIGURE 7.7

Wage bargaining coordination and government intervention in the OECD (average for the years 2000s). Coordination of wage bargaining: 5=strong coordination at national level, 1=strong fragmentation; Government intervention in wage bargaining: 5=strong intervention, 1=no intervention.

Source: Database on Institutional Characteristics of Trade Unions, Wage Setting, State (ICTWSS).

Bargaining over hours

- Real-world bargaining appears often to be about both wages and working time

Ω = wage income

T = time allocation

H = hours worked

$\Omega = wH$

Utility function of a worker is $v(\Omega, H)$

$e(H)$ = productivity of a worker

L = number of workers

Revenue of the firm

$$R[e(H)L] = [e(H)L]^\alpha / \alpha \quad \alpha \in [0, 1]$$

$\eta_H^e = He'(H)/e(H) > 0$ is the elasticity of worker productivity w.r.t. hours.

$e(H)/(H)$ = the productivity per hour. It increases with the number of hours if $\eta_H^e > 1$.

- Bargaining about the hourly wage and hours only

Union utility

$$V_s = \ell [\nu(\Omega, T - H)] + (1 - \ell) \nu(\bar{w}, T) \quad \ell = \text{Min}(1, L/N)$$

Firm profit

$$\pi = \frac{1}{\alpha} [e(H)L]^{\alpha} - \Omega L \quad (24)$$

Right-to-manage assumption

Firm determines employment from profit maximisation. w and H or equivalently Ω and H are taken as given.

Set $\partial\pi / \partial L = 0$ and solve for L :

$$L(\Omega, H) = [e(H)]^{\alpha/(1-\alpha)} \Omega^{1/(\alpha-1)} \quad (25)$$

If $L(\Omega, H) < N$, we can plug (25) into profit equation (24).

$$\pi(\Omega, H) = \left(\frac{1-\alpha}{\alpha} \right) \left[\frac{e(H)}{\Omega} \right]^{\alpha/(1-\alpha)}$$

Nash bargaining solution

If no agreement:

Employee gets $\nu(\bar{w}, T)$

Firm gets zero profit

$$\text{Max}_{\Omega, H} \left[\frac{L(\Omega, H)}{N} \right]^{\gamma} [\nu(\Omega, T - H) - \nu(\bar{w}, T)]^{\gamma} [\pi(\Omega, H)]$$

$$\text{s.t.} \quad L(\Omega, H) \leq N \quad \text{and} \quad H \leq \bar{H}$$

\bar{H} is legal constraint on hours (maximum hours allowed by legislation).

Interior solution

Take logs and differentiate w.r.t. Ω and H .

FOCs

$$\frac{\gamma \nu_1(\Omega, T - H)}{\nu(\Omega, T - H) - \nu(\bar{w}, T)} = \frac{\alpha(1 - \gamma) + \gamma}{(1 - \alpha)\Omega} \quad (26)$$

$$\frac{\gamma \nu_2(\Omega, T - H)}{\nu(\Omega, T - H) - \nu(\bar{w}, T)} = \frac{\alpha}{(1 - \alpha)} \frac{e'(H)}{e(H)} \quad (27)$$

Divide (26) by (27):

$$\begin{aligned} \frac{\nu_1(\Omega, T - H)}{\nu_2(\Omega, T - H)} &= \frac{[\alpha(1 - \gamma) + \gamma]}{(1 - \alpha)\Omega} \cdot \frac{(1 - \alpha)}{\alpha} \cdot \frac{e(H)}{e'(H)} = \\ &= \frac{[\alpha(1 - \gamma) + \gamma]}{\alpha} \cdot \frac{e(H)}{e'(H) \cdot H} \cdot \frac{H}{\Omega} = \frac{H}{\Omega} \frac{[\alpha(1 - \gamma) + \gamma]}{\alpha \eta_H^e} \end{aligned} \quad (28)$$

$$\eta_H^e = He'(H)/e(H)$$

Equation (28) defines the MRS between income and leisure as a function of the wage $w = \Omega/H$ and the elasticity of employee productivity w.r.t. H, η_h^e .

Assume Cobb-Douglas utility function:

$$\nu(\Omega, T - H) = (\Omega)^\mu (T - H)^{1-\mu} \quad \mu \in (0, 1)$$

Then:

$$\nu_1 = \mu \Omega^{\mu-1} (T - H)^{1-\mu}$$

$$\nu_2 = (1-\mu)(T - H)^{-\mu} \Omega^\mu$$

$$\frac{\nu_1}{\nu_2} = \frac{\mu}{1-\mu} \Omega^{-1} (T - H) = \frac{\mu}{1-\mu} \frac{(T - H)}{\Omega}$$

Assume that $e(H) = H$, then

$$e'(H) = 1 \quad \text{and} \quad \eta_H^e = e'(H) \cdot H / e(H) = 1.$$

(28) then simplifies to:

$$\frac{\mu}{1-\mu} \frac{(T - H)}{\Omega} = \frac{H}{\Omega} \left[\frac{\alpha(1-\gamma) + \gamma}{\alpha} \right]$$

$$H^* = \frac{\mu\alpha}{(1-\mu)[\gamma + \alpha(1-\gamma)] + \mu\alpha} \quad (29A)$$

Optimal number of hours

- is increasing in μ (the importance of income relative to leisure)
- is decreasing in union bargaining power γ
 - unions want low working time to get leisure and more workers employed
 - explanation of work sharing: reduction in hours to boost employment

Legal maximum of hours $\bar{H} < H^*$

Negotiated wage is then given by (26) with $H = \bar{H}$

With Cobb-Douglas preferences one obtains:

$$\Omega^\mu (T - \bar{H})^{1-\mu} = \frac{\gamma(1-\alpha) + \alpha}{\gamma(1-\mu)(1-\alpha) + \alpha} \nu(\bar{w}, T) \quad (\text{A})$$

RHS of (A) is a constant. Hence:

$$\Omega^\mu (T - \bar{H})^{1-\mu} = \text{constant}$$

$$\mu \ln \Omega + (1 - \mu) \ln(T - \bar{H}) = \text{constant}$$

Differentiate w.r.t. $d\ln\bar{H}$

$$\mu \cdot \frac{d\ln\Omega}{d\ln\bar{H}} + (1 - \mu) \frac{d\ln(T - \bar{H})}{d\ln\bar{H}} = 0$$

$$\mu \cdot \frac{d\ln\Omega}{d\ln\bar{H}} + (1 - \mu) \frac{d\ln(T - \bar{H})}{d\bar{H}} \cdot \frac{d\bar{H}}{d\ln\bar{H}} = 0$$

$$\mu \cdot \frac{d\ln\Omega}{d\ln\bar{H}} + (1 - \mu) \cdot \frac{(-1)}{T - \bar{H}} \cdot \bar{H} = 0$$

$$\frac{d\ln\Omega}{d\ln\bar{H}} = \eta_h^\Omega = \frac{\bar{H}(1 - \mu)}{(T - \bar{H}) \cdot \mu}$$

- **The elasticity of wage income w.r.t. hours, η_h^Ω , is positive.**
- **Hence wage income falls if hours fall.**
- **It falls more if hours are long to begin with.**

$$L(\Omega, H) = [e(H)]^{\alpha/(1-\alpha)} \Omega^{1/(\alpha-1)} \quad (25)$$

Assume again $e(H) = H$

$$L(\Omega, H) = H^{\alpha/(1-\alpha)} \Omega^{1/\alpha-1} \quad (\text{B})$$

- We want to know what happens to employment L if binding legal maximum \bar{H} is reduced.
 - direct effect from change in \bar{H}
 - indirect effect from induced change in wage income Ω .

Take logs of (B):

$$\ln L = \frac{\alpha}{1-\alpha} \ln \bar{H} + \frac{1}{\alpha-1} \ln \Omega$$

Differentiate w.r.t. $d \ln \bar{H}$

$$\frac{d \ln L}{d \ln \bar{H}} = \frac{\alpha}{1-\alpha} + \frac{1}{\alpha-1} \frac{d \ln \Omega}{d \ln \bar{H}}$$

We use:

$$\frac{d \ln \Omega}{d \ln \bar{H}} = \frac{\bar{H}(1-\mu)}{(T-\bar{H}) \cdot \mu}$$

$$\frac{d \ln L}{d \ln \bar{H}} = \frac{\alpha}{1-\alpha} + \frac{1}{\alpha-1} \cdot \frac{\bar{H}(1-\mu)}{(T-\bar{H}) \cdot \mu}$$

$$\frac{d \ln L}{d \ln \bar{H}} < 0 \quad \text{if} \quad \frac{\alpha}{1-\alpha} + \frac{1}{\alpha-1} \cdot \frac{\bar{H}(1-\mu)}{(T-\bar{H}) \cdot \mu} < 0$$

This is equivalent to $\bar{H} > \hat{H}$

$$\hat{H} = \frac{\mu\alpha}{(1-\mu) + \mu\alpha} T$$

Interpretation

- A reduction in working time raises employment only if $\bar{H} > \hat{H}$.
- From (29A) we have that \hat{H} is optimal hours for unions.

$$H^* = \frac{\mu\alpha}{(1-\mu)[\gamma + \alpha(1-\gamma)] + \mu\alpha} \quad (29A)$$

$$\gamma = 1 \Rightarrow$$

$$H^* = \frac{\mu\alpha}{(1-\mu) + \mu\alpha}$$

- A reduction in \bar{H} increases employment only down to the point where H reaches the trade union optimum.
- Further reductions lower employment.

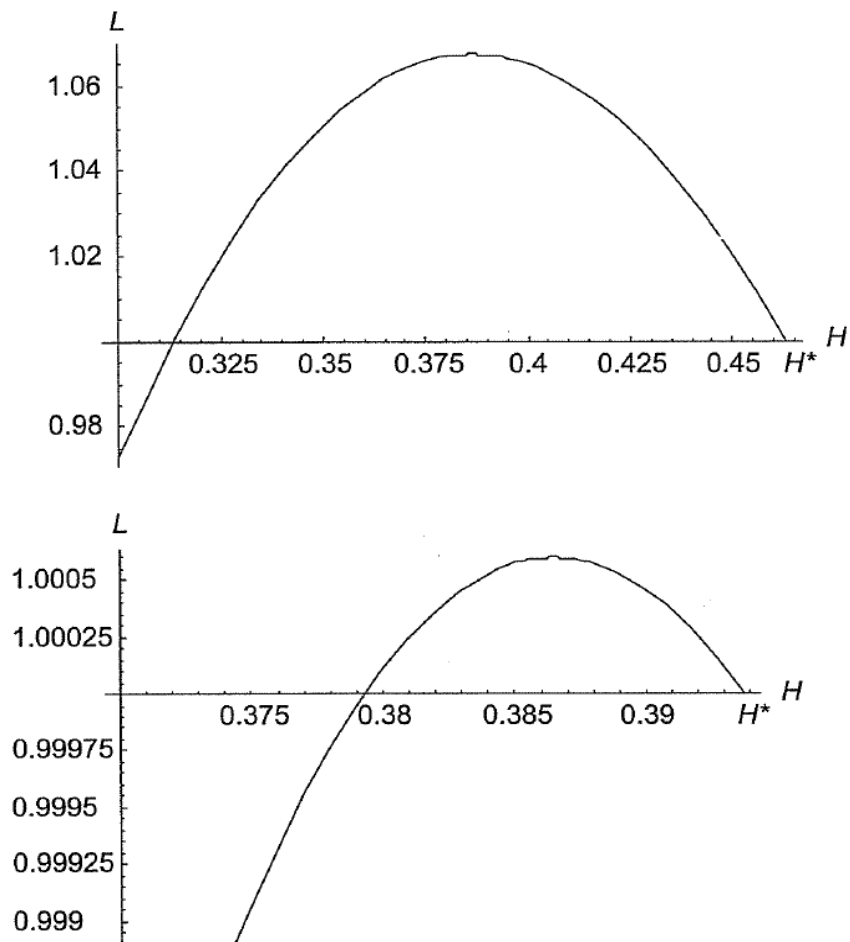


FIGURE 7.9

The impact of a reduction in the number of hours worked. The graph on the top corresponds to a value $\gamma = 0.1$ of bargaining power and the one on the bottom to $\gamma = 0.9$. The number of hours worked is given on the horizontal axis and stops at the negotiated number, H^* , which has a value of 0.463 (on the top) and 0.394 (on the bottom), knowing that the time allocation $T = 1$. The ratio between actual employment and its value for H^* is given on the vertical axis.

Insiders and outsiders

- Unions negotiate on behalf of insiders (the already employed those with a strong affiliation to the labour market)
- Unions do not negotiate on behalf of outsiders (the unemployed in general or the long-term unemployed)

An insider-outsider model

- L_O insiders
- The firm decides on how many insiders $L_I \leq L_O$ it wants to retain.
- It also decides on how many outsiders L_E it wants to hire.
- Revenue function $R(L_I + L_E)$
- The firm's profit: $\pi = R(L_I + L_E) - w(L_I + L_E)$
- Employment of insiders, L_I , and of outsiders, L_E , is found by maximising profits s. t. $L_I \leq L_O$ and $L_E \geq 0$.
- Define w_O by $R'(L_O) = w_O$.
- Define \tilde{L} as the employment level such that $R'(\tilde{L}) = w$, where w is the current wage.

Labour demand

$$L_I = \tilde{L} \text{ and } L_E = 0 \text{ if } w \geq w_O$$

$$L_I = L_O \text{ and } L_E = \tilde{L} - L_O \text{ if } w \leq w_O$$

If $w > w_O$ we have $L_I = \tilde{L} < L_O$, so some insiders are fired.

Wage bargaining

V_I = expected utility of an insider

$$V_I = \ell \nu(w) + (1 - \ell) \nu(\bar{w}) \quad \ell = \text{Min}(1, \tilde{L} / L_o)$$

\bar{w} = the reservation wage

$$\text{Max}_w \quad [\pi(w)]^{1-\gamma} \left\{ \ell [\nu(w) - \nu(\bar{w})] \right\}^\gamma$$

$$\text{with } \pi(w) = R(\tilde{L}) - w\tilde{L}$$

- Let w_1 be the solution when $\ell = \tilde{L} / L_o$ (interior solution with some unemployed insiders).
- The solution is the same as in the standard right-to-manage model but with $L_o = N$.

$$\frac{\nu(w_1) - \nu(\bar{w})}{w_1 \nu'(w_1)} = \frac{\gamma}{\gamma \eta_w^L + (1 - \gamma) \eta_w^\pi} \quad (10)$$

Solution with $\ell = 1$

- Set $\eta_w^L = 0$ in (10); employment of insiders cannot increase

$$\frac{\nu(w_2) - \nu(\bar{w})}{w_2 \nu'(w_2)} = \frac{\gamma}{(1 - \gamma) \eta_w^\pi}$$

Different solutions

B_1 = Nash bargaining product when $\tilde{L} > L_0$,
i.e. some employed outsiders

B_2 = Nash bargaining product when $\tilde{L} < L_0$,
i.e. some unemployed insiders

We have:

$$\frac{\partial B_1}{\partial w} > \frac{\partial B_2}{\partial w}$$

Larger gain from wage increase if only outsiders lose their jobs than if also insiders do.

Second-order conditions for a maximum

$$\frac{\partial^2 B_1}{\partial w^2} = \frac{\partial(\partial B_1 / \partial w)}{\partial w} < 0$$

$$\frac{\partial^2 B_2}{\partial w^2} = \frac{\partial(\partial B_2 / \partial w)}{\partial w} < 0$$

(1) **Interior solution with $w \leq w_0$ and $\tilde{L} \geq L_0$**

$$\frac{\partial B_1}{\partial w} = 0 \quad \frac{\partial B_2}{\partial w} < 0$$

(2) **Corner solution with $w = w_0$ and $\tilde{L} = L_0$**

$$\frac{\partial B_1}{\partial w} > 0 \quad \frac{\partial B_2}{\partial w} < 0$$

(3) **Interior solution with $w \geq w_0$ and $\tilde{L} \leq L_0$**

$$\frac{\partial B_1}{\partial w} > 0 \quad \frac{\partial B_2}{\partial w} = 0$$

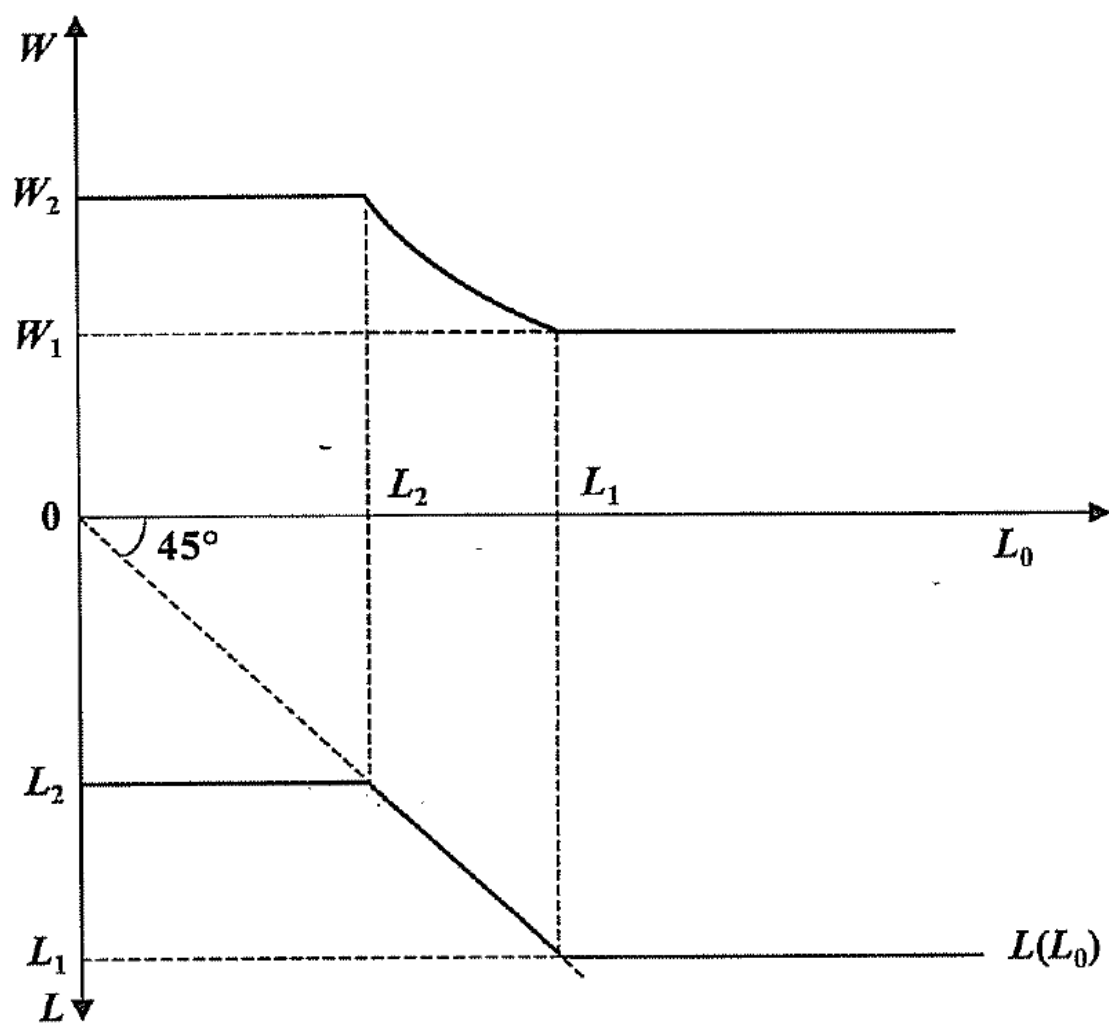


FIGURE 7.8

Wage and employment in the insiders/outsiders model.

Conclusion

- **A fall in the number of insiders results in an unchanged wage or in an increase in the wage**
- **Explanation of the persistence of unemployment**
- **No incentive to reduce the wage as the union does not care about the unemployed**
- **Empirical research has had problems finding that a reduction in lagged employment has a positive effect on the wage.**